

তৃতীয় অধ্যায়: বীজগাণিতিক রাশি

৩নং প্রশ্ন উত্তর (ক)

দেওয়া আছে,

$$x^2 - 3 = 2\sqrt{2}$$

$$\text{বা. } x^2 = 3 + 2\sqrt{2}$$

$$\text{বা. } x^2 = 2 + 2\sqrt{2} + 1$$

$$\text{বা. } x^2 = (\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot 1 + (1)^2$$

$$\text{বা. } x^2 = (\sqrt{2} + 1)^2$$

$$\text{বা. } x = \sqrt{2} + 1$$

$$\therefore x = \sqrt{2} + 1$$

Ans.

৩নং প্রশ্ন উত্তর (খ)

ক নং হতে প্রাপ্ত,

$$x = \sqrt{2} + 1$$

$$\therefore \frac{1}{x} = \frac{1}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2} - 1}{(\sqrt{2} + 1)(\sqrt{2} - 1)}$$

$$= \frac{\sqrt{2} - 1}{\{(\sqrt{2})^2 - (1)^2\}}$$

$$= \frac{\sqrt{2}-1}{2-1}$$

$$= \sqrt{2}-1$$

$$\therefore x + \frac{1}{x} = \sqrt{2}+1 + \sqrt{2}-1$$

$$= 2\sqrt{2}$$

প্রদত্ত রাশি : $x^4 + \frac{1}{x^4}$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \cdot x^2 \cdot \frac{1}{x^2}$$

$$= \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \quad \{^2-2$$

$$= \{2\sqrt{2}\}^2 - 2 \{^2-2$$

$$= \{4 \times 2\} - 2 \{^2-2$$

$$= \{8 - 2 \{^2-2$$

$$= (6)^2 - 2$$

$$= 36 - 2$$

$$= 34$$

A:

২নং প্রশ্ন উত্তর হা)

'খ' হতে পাই,

$$x + \frac{1}{x} = 2\sqrt{2}$$

$$\text{প্রাণে } x^4 + \frac{1}{x^4} = 34$$

$$\text{L.H.S} = x^5 + \frac{1}{x^5}$$

$$= \left(x^3 + \frac{1}{x^3}\right) \left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

$$= \left\{ \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \right\} \left\{ \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \right\} - \left(x + \frac{1}{x}\right)$$

$$= \left\{ (2\sqrt{2})^3 - 3 \times 2\sqrt{2} \right\} \left\{ (2\sqrt{2})^2 - 2 \right\} - 2\sqrt{2}$$

$$= \left\{ 8 \times 2\sqrt{2} - 6\sqrt{2} \right\} \left\{ 4 \times 2 - 2 \right\} - 2\sqrt{2}$$

$$= (16\sqrt{2} - 6\sqrt{2}) (8 - 2) - 2\sqrt{2}$$

$$= (10\sqrt{2} \times 6) - 2\sqrt{2}$$

$$= 60\sqrt{2} - 2\sqrt{2}$$

$$= 58\sqrt{2}$$

$$= \text{R.H.S}$$

(Proved)

২নং প্রশ্ন উত্তর (ক)

দেওয়া আছে, সংখ্যাটি a

\therefore সংখ্যাটির গুনাক্ষর বিপরীত সংখ্যা $\frac{1}{a}$

প্রকৃত্যে, $a + \frac{1}{a} = 2\sqrt{3}$

২নং প্রশ্ন উত্তর (খ)

ক নং হতে প্রাপ্ত

$$a + \frac{1}{a} = 2\sqrt{3}$$

প্রদত্ত রাশি : $a^3 + \frac{1}{a^3}$

$$= \left(a + \frac{1}{a}\right)^3 - 3a \cdot \frac{1}{a} \left(a + \frac{1}{a}\right)$$

$$= (2\sqrt{3})^3 - 3 \times 2\sqrt{3}$$

$$= (8 \times 3\sqrt{3}) - 6\sqrt{3}$$

$$= 24\sqrt{3} - 6\sqrt{3}$$

$$= 18\sqrt{3}$$

Ans

২নং প্রশ্ন উত্তর (গ)

'খ' নং হতে পাই,

$$a^3 + \frac{1}{a^3} = 18\sqrt{3}$$

$$\text{বা, } \frac{a^6 + 1}{a^3} = 18\sqrt{3}$$

$$\text{বা, } a^6 + 1 = 18\sqrt{3}a^3$$

$$\text{বা, } a^6 + 1 - 18\sqrt{3}a^3 = 0$$

$$\text{বা, } (a^3)^2 - 2 \cdot a^3 \cdot 9\sqrt{3} + (9\sqrt{3})^2 + 1 - (9\sqrt{3})^2 = 0$$

$$\text{বা, } (a^3 - 9\sqrt{3})^2 + 1 - 243 = 0$$

$$\text{বা, } (a^3 - 9\sqrt{3})^2 + 1 = 243$$

$$\text{বা, } (a^3 - 9\sqrt{3})^2 = 243 - 1$$

$$\text{বা, } (a^3 - 9\sqrt{3})^2 = 242$$

$$\text{বা, } (a^3 - 9\sqrt{3})^2 = (11\sqrt{2})^2$$

$$\text{বা, } (a^3 - 9\sqrt{3}) = (11\sqrt{2})$$

$$\text{বা, } a^3 = 9\sqrt{3} + 11\sqrt{2}$$

$$\text{বা, } a^3 = 3\sqrt{3} + 9\sqrt{2} + 6\sqrt{3} + 2\sqrt{2}$$

$$\text{বা, } a^3 = (\sqrt{3})^3 + 3 \cdot (\sqrt{3})^2 \cdot \sqrt{2} + 3 \cdot (\sqrt{3}) \cdot (\sqrt{2})^2 + (\sqrt{2})^3$$

$$\text{বা, } a^3 = (\sqrt{3} + \sqrt{2})^3$$

$$\text{বা. } a = (\sqrt{3} + \sqrt{2})$$

$$\therefore a = \sqrt{3} + \sqrt{2}$$

$$\therefore a = \sqrt{3} + \sqrt{2}$$

(Proved)

৩ নং প্রশ্নের উত্তর (ক)

প্রদত্ত রাশি : $a^4 + a^2 + 1$

$$= (a^2)^2 + 2a^2 \cdot 1 + (1)^2 - a^2$$

$$= (a^2 + 1)^2 - a^2$$

$$= (a^2 + 1 + a)(a^2 + 1 - a)$$

$$= (a^2 + a + 1)(a^2 - a + 1)$$

A:

৩ নং সমস্যা সম্বন্ধে (খ)

দেওয়া আছে,

$$x^2 - 2x + 1 = 0$$

$$\text{বা. } x^2 + 1 = 2x$$

$$\text{বা. } \frac{x^2 + 1}{x} = \frac{2x}{x}$$

$$\text{বা. } \frac{x^2}{x} + \frac{1}{x} = 2$$

$$\text{বা. } x + \frac{1}{x} = 2$$

$$\therefore x + \frac{1}{x} = 2$$

$$\text{L.H.S} = x^2 + \frac{1}{x^2}$$

$$= \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x}$$

$$= (2)^2 - 2$$

$$= 4 - 2$$

$$= 2$$

$$\text{R.H.S} = x^4 + x^{-4}$$

$$= x^4 + \frac{1}{x^4}$$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2$$

$$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \cdot x^2 \cdot \frac{1}{x^2}$$

$$\begin{aligned}
 &= \left\{ \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \right\}^{\frac{1}{2}} = 2 \\
 &= \left\{ (2)^2 - 2 \right\}^{\frac{1}{2}} = 2 \\
 &= (4-2)^{\frac{1}{2}} = 2 \\
 &= (2)^{\frac{1}{2}} = 2 \\
 &= 4-2 \\
 &= 2
 \end{aligned}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

(Proved)

৩ নং প্রশ্নের উত্তর হল

ক) হতে পাই,

$$x + \frac{1}{x} = 2$$

আমরা জানি,

$$\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x}$$

$$\therefore \left(x - \frac{1}{x}\right) = \sqrt{(2)^2 - 4}$$

$$= \sqrt{4-4}$$

$$= \sqrt{0}$$

$$= 0$$

প্রদত্ত রাশি : $x^5 - \frac{1}{x^3}$

$$= (x^3 - \frac{1}{x^3})(x^2 + \frac{1}{x^2}) - (x + \frac{1}{x})$$

$$= \left\{ (x - \frac{1}{x})^3 + 3 \cdot x \cdot \frac{1}{x} (x - \frac{1}{x}) \right\}$$

$$\left\{ (x + \frac{1}{x})^2 - 2 \cdot x \cdot \frac{1}{x} \right\} - (x + \frac{1}{x})$$

$$= \left\{ (0)^3 + 3 \times 0 \right\} \left\{ (2)^2 - 2 \right\} - 0$$

$$= \left\{ 0 + 0 \right\} \left\{ 4 - 2 \right\} - 0$$

$$= 0 \times 2 - 0$$

$$= 0 - 0$$

$$= 0$$

A:

৪ নং প্রশ্ন উত্তর (ক)

দেওয়া আছে,

$$b^2 - 2\sqrt{6}b + 1 = 0$$

$$\text{বা, } b^2 + 1 = 2\sqrt{6}b$$

$$\text{বা, } \frac{b^2 + 1}{b} = \frac{2\sqrt{6}b}{b}$$

$$\text{বা, } \frac{b^2}{b} + \frac{1}{b} = 2\sqrt{6}$$

$$\text{বা, } b + \frac{1}{b} = 2\sqrt{6}$$

$$\therefore b + \frac{1}{b} = 2\sqrt{6} \quad (\text{দেখানো হল})$$

৪ নং পুস্তক উত্তর (খ)

(ক) হতে পাই,

$$b + \frac{1}{b} = 2\sqrt{6}$$

$$\text{বা, } (b + \frac{1}{b})^2 = (2\sqrt{6})^2$$

$$\text{বা, } (b + \frac{1}{b})^2 = (4 \times 6)$$

$$\text{বা, } (b - \frac{1}{b})^2 + 4 \cdot b \cdot \frac{1}{b} = 24$$

$$\text{বা, } (b - \frac{1}{b})^2 = 24 - 4$$

$$\text{বা, } (b - \frac{1}{b})^2 = 20$$

$$\text{বা, } (b - \frac{1}{b})^2 = (\sqrt{20})^2$$

$$\text{বা, } (b - \frac{1}{b}) = (2\sqrt{5})^2$$

$$\therefore b - \frac{1}{b} = 2\sqrt{5}$$

প্রদত্ত রাশি: $\frac{1}{b^3} (b^6 - 1)$

$$= b^3 - \frac{1}{b^3}$$

$$= (b - \frac{1}{b})^3 + 3 \cdot b \cdot \frac{1}{b} (b - \frac{1}{b})$$

$$= (2\sqrt{5})^3 + 3 \times 2\sqrt{5}$$

$$= 40\sqrt{5} + 6\sqrt{5}$$

$$= 46\sqrt{5}$$

৪ নং পূর্ণা টেক্সট (গ)

ক) প্রমাণ করুন,

$$b + \frac{1}{b} = 2\sqrt{6}$$

$$L.H.S = b^5 + \frac{1}{b^5}$$

$$= (b^3 + \frac{1}{b^3}) (b^2 + \frac{1}{b^2}) - (b + \frac{1}{b})$$

$$= \left\{ (b + \frac{1}{b})^3 - 3 \cdot b \cdot \frac{1}{b} (b + \frac{1}{b}) \right\} \left\{ (b + \frac{1}{b})^2 - 2 \cdot b \cdot \frac{1}{b} \right\} - (b + \frac{1}{b})$$

$$= \left\{ (2\sqrt{6})^3 - 3 \times 2\sqrt{6} \right\} \left\{ (2\sqrt{6})^2 - 2 \right\} - 2\sqrt{6}$$

$$= (48\sqrt{6} - 6\sqrt{6})(24 - 2) - 2\sqrt{6}$$

$$= (42\sqrt{6} \times 22) - 2\sqrt{6}$$

$$= 924\sqrt{6} - 2\sqrt{6}$$

$$= 922\sqrt{6}$$

$$= R.H.S$$

(Proved)

৬ নং প্রশ্ন উত্তর (ক)

$$x^2 = 5 + 2\sqrt{6}$$

$$\text{বা. } x^2 = (\sqrt{3})^2 + 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2$$

$$\text{বা. } x^2 = (\sqrt{3} + \sqrt{2})^2$$

$$\text{বা. } x = \sqrt{3} + \sqrt{2}$$

$$\therefore x = \sqrt{3} + \sqrt{2}$$

A.

৭ নং প্রশ্ন উত্তর (খ)

ক নং সঠিক পাই,

$$x = \sqrt{3} + \sqrt{2}$$

$$\therefore \frac{1}{x} = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{3 - 2}$$

$$= \sqrt{3} - \sqrt{2}$$

$$= \sqrt{3} - \sqrt{2}$$

$$\begin{aligned} \therefore x + \frac{1}{x} &= \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2} \\ &= 2\sqrt{3} \end{aligned}$$

$$\text{L.H.S} = \frac{x^8 + 1}{x^4}$$

$$= \frac{x^8}{x^4} + \frac{1}{x^4}$$

$$= x^4 + \frac{1}{x^4}$$

$$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2$$

$$= \left\{ \left(x^2 + \frac{1}{x^2}\right)^2 - 2 \cdot x^2 \cdot \frac{1}{x^2} \right\}$$

$$= \left\{ \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \right\}^2 - 2$$

$$= \left\{ (2\sqrt{3})^2 - 2 \right\}^2 - 2$$

$$= \left\{ (4 \times 3) - 2 \right\}^2 - 2$$

$$= (12 - 2)^2 - 2$$

$$= (10)^2 - 2$$

$$= 100 - 2$$

$$= 98$$

$$= \text{R.H.S}$$

(Proved)

৩ নং প্রশ্নে দেখে গে।

দেওয়া আছে,

$$a+b=m$$

$$a^2+b^2=n \quad [\because c=0]$$

$$a^3+b^3=p^3$$

$$L.H.S = m^3 + 2p^3$$

$$= (a+b)^3 + 2(a^3+b^3)$$

$$= a^3 + 3a^2b + 3ab^2 + b^3 + 2a^3 + 2b^3$$

$$= 3a^3 + 3a^2b + 3ab^2 + 3b^3$$

$$= 3(a^3 + a^2b + ab^2 + b^3)$$

$$= 3\{a^2(a+b) + b^2(a+b)\}$$

$$= 3(a^2+b^2)(a+b)$$

$$= 3nm$$

$$= 3mn$$

$$= R.H.S$$

(shown)

৬নং প্রশ্নে উত্তর (ক)

দেওয়া আছে,

$$p^4 = 5 + 2\sqrt{6}$$

$$= 3 + 2\sqrt{6} + 2$$

$$= (\sqrt{3})^4 + 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^4$$

$$= (\sqrt{3} + \sqrt{2})^4$$

$$\therefore p^4 = (\sqrt{3} + \sqrt{2})^4$$

$$\text{সি. } p = \sqrt{3} + \sqrt{2}$$

$$\text{প্রকৃ } \frac{1}{p} = \frac{1}{\sqrt{3} + \sqrt{2}}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{3 - 2}$$

$$= \sqrt{3} - \sqrt{2}$$

$$P - \frac{1}{P} = \sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2}$$

$$= 2\sqrt{2}$$

সি.

৬ নং প্রশ্নে উত্তর (খ)

দেওয়া আছে,

$$a^3 + a^{-3} = 18\sqrt{3}$$

$$\text{বা. } (a^3 + a^{-3})^2 = (18\sqrt{3})^2 \text{ [বর্গ করে]}$$

$$\text{বা. } (a^3 - a^{-3})^2 + 4 \cdot a^3 \cdot a^{-3} = 972$$

$$\text{বা. } (a^3 - a^{-3})^2 + 4 \cdot a^3 \cdot \frac{1}{a^3} = 972$$

$$\text{বা. } (a^3 - a^{-3})^2 + 4 = 972$$

$$\text{বা. } (a^3 - a^{-3})^2 = 972 - 4$$

$$\text{বা. } (a^3 - a^{-3})^2 = 968$$

$$\text{বা. } (a^3 - a^{-3}) = 22\sqrt{2}$$

$$\therefore a^3 - a^{-3} = 22\sqrt{2}$$

$$\text{প্রথম, } a^3 + a^{-3} + a^3 - a^{-3} = 18\sqrt{3} + 22\sqrt{2}$$

$$\text{বা. } 2a^3 = 18\sqrt{3} + 22\sqrt{2}$$

$$\text{বা. } a^3 = \frac{18\sqrt{3} + 22\sqrt{2}}{2}$$

$$\text{বা, } a^3 = \frac{18\sqrt{3}}{2} + \frac{22\sqrt{2}}{2}$$

$$\text{বা, } a^3 = 9\sqrt{3} + 11\sqrt{2}$$

$$\text{বা, } a^3 = 3\sqrt{3} + 9\sqrt{2} + 6\sqrt{3} + 2\sqrt{2}$$

$$\text{বা, } a^3 = (\sqrt{3})^3 + 3(\sqrt{3})^2 \cdot \sqrt{2} + 3\sqrt{3} \cdot (\sqrt{2})^2 + (\sqrt{2})^3$$

$$\text{বা, } a^3 = (\sqrt{3} + \sqrt{2})^3$$

$$\text{বা, } a = \sqrt{3} + \sqrt{2}$$

$$\therefore a = \sqrt{3} + \sqrt{2}$$

(প্রমাণিত)

৬ নং প্রশ্নের উত্তর (গ)

ক) থেকে পাই,

$$p = \sqrt{3} + \sqrt{2}$$

$$\therefore \frac{1}{p} = \sqrt{3} - \sqrt{2}$$

$$\therefore p + \frac{1}{p} = (\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}) \\ = 2\sqrt{3}$$

$$\text{L.H.S} = \frac{p^{10} + 1}{p^5}$$

$$= \frac{p^{10}}{p^5} + \frac{1}{p^5}$$

$$= p^3 + \frac{1}{p^3}$$

$$= \left(p^2 + \frac{1}{p}\right) \left(p^3 + \frac{1}{p^3}\right) - \left(p + \frac{1}{p}\right)$$

$$= \left\{ \left(p + \frac{1}{p}\right)^2 - 2 \cdot \frac{1}{p} \right\} \left\{ \left(p + \frac{1}{p}\right)^3 - 3 \cdot p \cdot \frac{1}{p} \left(p + \frac{1}{p}\right) \right\} - \left(p + \frac{1}{p}\right)$$

$$= \left\{ (2\sqrt{3})^2 - 2 \right\} \left\{ (2\sqrt{3})^3 - 3 \times 2\sqrt{3} \right\} - 2\sqrt{3}$$

$$= \left\{ (4 \times 3) - 2 \right\} \left\{ (8 \times 3\sqrt{3}) - 6\sqrt{3} \right\} - 2\sqrt{3}$$

$$= (12 - 2) \left\{ 24\sqrt{3} - 6\sqrt{3} \right\} - 2\sqrt{3}$$

$$= (10 \times 18\sqrt{3}) - 2\sqrt{3}$$

$$= 180\sqrt{3} - 2\sqrt{3}$$

$$= 178\sqrt{3}$$

$$= R.H.S$$

(Proved)

৭নং প্রশ্নে উত্তর (ক)

দেওয়া আছে,

$$x^2 + \frac{1}{x^2} = 10$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 = 2 \cdot x \cdot \frac{1}{x} = 10$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 - 2 = 10$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 = 10 + 2$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 = 12$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 = \sqrt{12}$$

$$\text{বা, } x + \frac{1}{x} = \sqrt{4 \times 3}$$

$$\text{বা, } x + \frac{1}{x} = 2\sqrt{3}$$

$$\therefore x + \frac{1}{x} = 2\sqrt{3}$$

∴

৭নং প্রশ্ন সমাধান

'ক' হতে পাই,

$$x + \frac{1}{x} = 2\sqrt{3}$$

আমরা জানি,

$$\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x}$$

$$\begin{aligned} \therefore x - \frac{1}{x} &= \sqrt{(2\sqrt{3})^2 - 4} \\ &= \sqrt{12 - 4} \\ &= \sqrt{8} \\ &= \sqrt{4 \times 2} \\ &= 2\sqrt{2} \end{aligned}$$

$$\text{L.H.S} = \frac{x^8 - 1}{x^4}$$

$$= \frac{x^8}{x^4} - \frac{1}{x^4}$$

$$= x^4 - \frac{1}{x^4}$$

$$= (x^4)^2 - \left(\frac{1}{x^4}\right)^2$$

$$= \left(x^2 + \frac{1}{x^2}\right) \left(x^2 - \frac{1}{x^2}\right)$$

$$= \left\{ \left(x + \frac{1}{x} \right)^2 - 2 \cdot x \cdot \frac{1}{x} \right\} \left(x + \frac{1}{x} \right) \left(x - \frac{1}{x} \right)$$

$$= \left\{ (2\sqrt{3})^2 - 2 \right\} \left\{ (2\sqrt{3}) \times 2\sqrt{2} \right\}$$

$$= (12 - 2) \times 4\sqrt{6}$$

$$= 10 \times 4\sqrt{6}$$

$$= 40\sqrt{6}$$

$$= \text{R.H.S}$$

(প্রমাণিত)

৭নং প্রশ্ন উত্তর (গ)

প্রথমে, $x^2 + \frac{1}{x^2} = 10$ [২য় নং হতে প্রাপ্ত]

২য়ং $(x - \frac{1}{x}) = 2\sqrt{2}$ [২য় নং হতে প্রাপ্ত]

প্রদত্ত রাশিঃ $x^5 - \frac{1}{x^5}$

$$= \left(x^2 + \frac{1}{x^2} \right) \left(x^3 - \frac{1}{x^3} \right) - \left(x + \frac{1}{x} \right)$$

$$= 10 \times \left\{ \left(x - \frac{1}{x} \right)^3 + 3 \cdot x \cdot \frac{1}{x} \left(x - \frac{1}{x} \right) \right\} - \left(x + \frac{1}{x} \right)$$

$$= 10 \times \left\{ (2\sqrt{2})^3 + 3 \times 2\sqrt{2} \right\} - 2\sqrt{2}$$

$$= 10 \times \left\{ (8 \times 2\sqrt{2}) + 6\sqrt{2} \right\} - 2\sqrt{2}$$

$$\begin{aligned}
 &= 10 \times (16\sqrt{2} + 6\sqrt{2}) - 2\sqrt{2} \\
 &= 10 \times 22\sqrt{2} - 2\sqrt{2} \\
 &= 220\sqrt{2} - 2\sqrt{2} \\
 &= 218\sqrt{2}
 \end{aligned}$$

A:

চক্রে প্রয়োগ করে (ক)

দেওয়া আছে,

$$P^2 = 7 + 4\sqrt{3}$$

$$\text{বা. } P^2 = 4 + 4\sqrt{3} + 3$$

$$\text{বা. } P^2 = (2)^2 + 2 \cdot 2\sqrt{3} + (\sqrt{3})^2$$

$$\text{বা. } P^2 = (2 + \sqrt{3})^2$$

$$\text{বা. } P = 2 + \sqrt{3}$$

$$\therefore P = 2 + \sqrt{3}$$

A:

চ নং প্রশ্ন উত্তর(খ)

'ক' হতে পাই,

$$p = 2 + \sqrt{3}$$

$$\therefore \frac{1}{p} = \frac{1}{2 + \sqrt{3}}$$

$$= \frac{2 - \sqrt{3}}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \frac{2 - \sqrt{3}}{\{2\}^2 - \{\sqrt{3}\}^2}$$

$$= \frac{2 - \sqrt{3}}{4 - 3}$$

$$= 2 + \sqrt{3}$$

$$\therefore p - \frac{1}{p} = 2 + \sqrt{3} - 2 + \sqrt{3}$$

$$= 2\sqrt{3}$$

সুতরাং : $\frac{p^6 - 1}{p^3}$

$$= \frac{p^6}{p^3} - \frac{1}{p^3}$$

$$\begin{aligned}
 &= \left(p^3 - \frac{1}{p^3}\right) \\
 &= \left(p - \frac{1}{p}\right)^3 + 3p \cdot \frac{1}{p} \left(p - \frac{1}{p}\right) \\
 &= (2\sqrt{3})^3 + 3 \times 2\sqrt{3} \\
 &= (8 \times 3\sqrt{3}) + 6\sqrt{3} \\
 &= 24\sqrt{3} + 6\sqrt{3} \\
 &= 30\sqrt{3}
 \end{aligned}$$

A:

চলুন প্রমাণ করি (১)

'ক' হতে পাই,

$$p = 2 + \sqrt{3}$$

প্রমাণ,

'খ' হতে পাই,

$$\frac{1}{p} = \frac{1(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$= \frac{2 - \sqrt{3}}{4 - 3}$$

$$= 2 - \sqrt{3}$$

$$\begin{aligned}
 \therefore p + \frac{1}{p} &= 2 + \sqrt{3} + 2 - \sqrt{3} \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 \text{L.H.S} &= p^5 + \frac{1}{p^5} \\
 &= \left(p^3 + \frac{1}{p^3}\right) \left(p^2 + \frac{1}{p^2}\right) - \left(p + \frac{1}{p}\right) \\
 &= \left\{ \left(p + \frac{1}{p}\right)^3 - 3 \cdot p \cdot \frac{1}{p} \left(p + \frac{1}{p}\right) \right\} \left\{ \left(p + \frac{1}{p}\right)^2 - 2 \cdot p \cdot \frac{1}{p} \right\} \\
 &\quad - \left(p + \frac{1}{p}\right) \\
 &= \left\{ (4)^3 - 3 \times 4 \right\} \left\{ (4)^2 - 2 \right\} - 4 \\
 &= \left\{ 64 - 12 \right\} \left\{ 16 - 2 \right\} - 4 \\
 &= (52 \times 14) - 4 \\
 &= 728 - 4 \\
 &= 724 \\
 &= \text{R.H.S}
 \end{aligned}$$

(প্রমাণিত)

২৩ নং প্রশ্নের উত্তর (ক)

দেওয়া আছে,

$$x = 3 + 2\sqrt{2}$$

$$\therefore \frac{1}{x} = \frac{1}{3 + 2\sqrt{2}}$$

$$= \frac{3 - 2\sqrt{2}}{(3 + 2\sqrt{2})(3 - 2\sqrt{2})}$$

$$= \frac{3 - 2\sqrt{2}}{(3)^2 - (2\sqrt{2})^2}$$

$$= \frac{3-2\sqrt{2}}{3-2}$$

$$= 3-2\sqrt{2}$$

২০ নং পূর্ণ্য উত্তর (খ)

দেখা আছে,

$$x = 3+2\sqrt{2}$$

ক'নং হতে প্রাপ্ত,

$$\frac{1}{x} = 3-2\sqrt{2}$$

$$\therefore x + \frac{1}{x} = 3+2\sqrt{2} + 3-2\sqrt{2}$$

$$= 6$$

প্রদত্ত রাশি : $x^6 + \frac{1}{x^6}$

$$= (x^3)^2 + \left(\frac{1}{x^3}\right)^2$$

$$= \left(x^3 + \frac{1}{x^3}\right)^2 - 2 \cdot x^3 \cdot \frac{1}{x^3}$$

$$= \left\{ \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) \right\}^2 - 2$$

$$= \left\{ (6)^3 - 3 \times 6 \right\}^2 - 2$$

$$= (216 - 18)^2 - 2$$

$$= (198)^2 - 2$$

$$= 39204 - 2$$

$$= 39202$$

A.

২০ নং প্রশ্ন উত্তর (গ)

দেওয়া আছে,

$$x = 3 + 2\sqrt{2}$$

$$\text{বা. } x = 3 + 2\sqrt{2} + 1$$

$$\text{বা. } x = (\sqrt{2})^2 + 2\sqrt{2} \cdot 1 + (1)^2$$

$$\text{বা. } x = (\sqrt{2} + 1)^2$$

$$\text{বা. } \sqrt{x} = \sqrt{(\sqrt{2} + 1)^2}$$

$$\text{বা. } \sqrt{x} = (\sqrt{2} + 1)$$

$$\therefore \sqrt{x} = (\sqrt{2} + 1)$$

$$\therefore \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2} + 1}$$

$$= \frac{\sqrt{2} - 1}{(\sqrt{2} + 1)(\sqrt{2} - 1)}$$

$$= \frac{\sqrt{2} - 1}{(\sqrt{2})^2 - (1)^2}$$

$$= \frac{\sqrt{2} - 1}{2 - 1}$$

$$= \sqrt{2} - 1$$

$$\therefore \sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{2+1} - \sqrt{2-1}$$

$$= 2$$

$$\text{L.H.S} = (\sqrt{x})^3 - \left(\frac{1}{\sqrt{x}}\right)^3$$

$$= (\sqrt{x} - \frac{1}{\sqrt{x}})^3 + 3 \cdot \sqrt{x} \cdot \frac{1}{\sqrt{x}} (\sqrt{x} - \frac{1}{\sqrt{x}})$$

$$= (2)^3 + 3 \times 2$$

$$= 8 + 6$$

$$= 14$$

$$= \text{R.H.S}$$

(প্রমাণিত)

৩৩ নং প্রশ্নের উত্তর (ক)

দেওয়া আছে,

$$x + y = \sqrt{3}$$

$$\text{প্রকং, } x^2 - y^2 = \sqrt{6}$$

$$\therefore x^2 - y^2 = \sqrt{6}$$

$$\text{বা, } (x+y)(x-y) = \sqrt{6}$$

$$\text{বা, } \sqrt{3}(x-y) = \sqrt{3 \cdot 2}$$

$$\text{বা, } (x-y) = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{3}}$$

$$\therefore x - y = \sqrt{2}$$

প্ৰদত্ত রাশিঃ $x^4 - y^4$

$$= \left(\frac{x+y}{2}\right)^4 - \left(\frac{x-y}{2}\right)^4$$

$$= \left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{\sqrt{2}}{2}\right)^4$$

$$= \frac{3}{4} - \frac{2}{4}$$

$$= \frac{3-2}{4}$$

$$= \frac{1}{4}$$

∴

১০নং প্ৰশ্নৰ উত্তৰ হৈছে

দেওয়া আছে,

$$x+y = \sqrt{3}$$

ক'নো হতে প্ৰাপ্ত,

$$xy = \frac{1}{4}$$

$$\text{L.H.S} = x^3 + y^3 + \frac{\sqrt{27}}{4}$$

$$= (x+y)^3 - 3xy(x+y) + \frac{\sqrt{27}}{4}$$

$$= (\sqrt{3})^3 - 3 \cdot \frac{1}{4} \cdot \sqrt{3} + \frac{\sqrt{9 \cdot 3}}{4}$$

$$= 3\sqrt{3} - \frac{3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}$$

$$= 3\sqrt{3}$$

$$= R.H.S$$

(shown)

২২ নং পূর্ণা উত্তর (৬)

ক) থেকে পাই,

$$x - y = \sqrt{2}$$

$$\text{একং } xy = \frac{1}{4}$$

একং,

$$x^2 + y^2 = (x - y)^2 + 2xy$$

$$= (\sqrt{2})^2 + 2 \cdot \frac{1}{4}$$

$$= 4 + \frac{1}{2}$$

$$= \frac{4+1}{2}$$

$$= \frac{5}{2}$$

প্রদত্ত রাশি : $16xy(x^2 + y^2)$

$$= 16 \times \frac{1}{4} \cdot \frac{5}{2}$$

$$= 2 \times 5$$

$$= 10$$

∴

১২ নং পূর্ণ উত্তর (ক)

দেওয়া আছে,

$$a+b+c=0$$

$$\therefore a+b=-c$$

$$\text{L.H.S} = a^3+b^3+c^3$$

$$= (a+b)^3 - 3ab(a+b) + c^3$$

$$= (-c)^3 - 3ab(-c) + c^3$$

$$= -c^3 + 3abc + c^3$$

$$= 3abc$$

$$= \text{R.H.S}$$

(Proved)

১২ নং পূর্ণ উত্তর (খ)

দেওয়া আছে,

$$a+b+c=10$$

$$a^2+b^2+c^2=38$$

প্রমাণে, $a+b+c=10$

বা. $(a+b+c)^2 = (10)^2$ [বর্গ করে]

$$\text{বা, } a^2 + b^2 + c^2 + 2(ab + bc + ca) = 100$$

$$\text{বা, } 38 + 2(ab + bc + ca) = 100$$

$$\text{বা, } 2(ab + bc + ca) = 100 - 38$$

$$\text{বা, } 2(ab + bc + ca) = 62$$

$$\text{বা, } ab + bc + ca = \frac{62}{2}$$

$$\therefore ab + bc + ca = 31$$

$$\text{প্রদত্ত রাশি: } (a-b)^2 + (b-c)^2 + (c-a)^2$$

$$= a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2$$

$$= a^2 + b^2 - 2ab + b^2 + c^2 + 2bc + c^2 + a^2 - 2ca$$

$$= 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca$$

$$= 2(a^2 + b^2 + c^2) - 2(ab + bc + ca)$$

$$= (2 \times 38) - (31 \times 2)$$

$$= 76 - 62$$

$$= 14$$

A:

৩২নং প্রশ্নে উত্তর (৬)

দেওয়া আছে,

$$a+b+c=0$$

$$\text{L.H.S} = \frac{(b+c)^2}{6bc} + \frac{(c+a)^2}{6ac} + \frac{(a+b)^2}{6ab}$$

$$= \frac{(b+c+a-a)^2}{6bc} + \frac{(c+a+b-b)^2}{6ac} +$$

$$\frac{(a+b+c-c)^2}{6ab}$$

$$= \frac{(0-a)^2}{6bc} + \frac{(a-b)^2}{6ac} + \frac{(0-c)^2}{6ab}$$

$$= \frac{a^2}{6bc} + \frac{b^2}{6ac} + \frac{c^2}{6ab}$$

$$= \frac{a^3 + b^3 + c^3}{6abc}$$

$$= \frac{(a+b)^3 + 3ab(a+b) + c^3}{6abc}$$

$$= \frac{(-c)^3 - 3ab(-c) + c^3}{6abc}$$

$$= \frac{-c^3 + 3abc + c^3}{6abc}$$

$$= \frac{3abc}{6abc}$$

$$= \frac{1}{2}$$

$$= R.H.S$$

(Proved)

২৩ নং প্রশ্নের জবাব (ক)

দেওয়া আছে,

$$a^4 + a^2b^2 + b^4 = 21$$

$$\text{বা. } (a^2)^2 + 2a^2b^2 + (b^2)^2 - a^2b^2 = 21$$

$$\text{বা. } (a^2 + b^2)^2 - (ab)^2 = 21$$

$$\text{বা. } (a^2 + ab + b^2)(a^2 - ab + b^2) = 21$$

$$\text{বা. } 7(a^2 - ab + b^2) = 21 \quad [\because a^2 + ab + b^2 = 3]$$

$$\text{বা. } a^2 - ab + b^2 = \frac{21}{7}$$

$$\therefore a^2 - ab + b^2 = 3$$

∴

১৩ নং প্রশ্ন দেয় (হা)

দেওয়া আছে,

$$a^2 + ab + b^2 = 7$$

ক নং হতে প্রাপ্ত,

$$a^2 - ab + b^2 = 3$$

(i) + (ii) যোগ করে পাই,

$$a^2 + ab + b^2 = 7$$

$$a^2 - ab + b^2 = 3$$

$$\hline 2a^2 \qquad \qquad \qquad + 2b^2 = 10$$

$$\text{বা. } 2(a^2 + b^2) = 10$$

$$\text{বা. } (a^2 + b^2) = \frac{10}{2}$$

$$\text{বা. } a^2 + b^2 = 5$$

$$\therefore a^2 + b^2 = 5$$

$a^2 + b^2$ এর মান (i) নং এ বসাই,

$$a^2 + ab + b^2 = 7$$

$$\text{বা. } a^2 + b^2 + ab = 7$$

$$\text{বা. } 5 + ab = 7$$

$$\text{বা. } ab = 7 - 5$$

$$\therefore ab = 2$$

$$L.H.S = 8ab(a^2 + b^2)$$

$$= 8 \times 2 \times 5$$

$$= 80$$

$$= R.H.S$$

(Proved)

২৩নং প্রশ্নে উত্তর (গ)

প্রদত্ত হতে পাই,

$$a^2 + b^2 = 5$$

$$ab = 2$$

আমরা জানি,

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\therefore (a+b) = \sqrt{a^2 + b^2 + 2ab}$$

$$= \sqrt{5 + 2 \times 2}$$

$$= \sqrt{5 + 4}$$

$$= \sqrt{9}$$

$$\therefore (a+b) = 3 \quad \text{--- (i)}$$

প্রাথমিক,

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\text{বা. } a-b = \sqrt{a^2 + b^2 - 2ab}$$

$$= \sqrt{5 - 2 \times 2}$$

$$= \sqrt{5-4}$$

$$= \sqrt{1}$$

$$\therefore (a-b) = 1 \quad \text{--- (ii)}$$

(i)+(ii) করে পাই,

$$a+b = 3$$

$$a-b = 1$$

$$\hline 2a = 4$$

$$\text{বা. } a = \frac{4}{2}$$

$$\therefore a = 2$$

(i) নং এ a এর মান বসাই,

$$a+b = 3$$

$$\text{বা. } 2+b = 3$$

$$\text{বা. } b = 3-2$$

$$\text{বা. } b = 1$$

$$\therefore b = 1$$

২০ নং প্রশ্নের (ক)

দেওয়া আছে,

$$x + \frac{1}{x} = 6$$

$$\begin{aligned} \text{প্রমাণ, } (x - \frac{1}{x})^2 &= (x + \frac{1}{x})^2 - 4 \cdot x \cdot \frac{1}{x} \\ &= (6)^2 - 4 \\ &= 36 - 4 \\ &= 32 \end{aligned}$$

∴

২০ নং প্রশ্নের (খ)

দেওয়া আছে,

$$x + \frac{1}{x} = 6$$

$$\begin{aligned} \text{L.H.S} &= x^3 + \frac{1}{x^3} \\ &= (x + \frac{1}{x})^3 - 3 \cdot x \cdot \frac{1}{x} (x + \frac{1}{x}) \\ &= (6)^3 - 3 \times 6 \\ &= 216 - 18 \\ &= 198 \\ &= \text{R.H.S} \end{aligned}$$

(shown)

৩০ নং প্রশ্ন উত্তর (১)

দেওয়া আছে,

$$x + \frac{1}{x} = 6$$

$$\text{L.H.S} = x^5 + \frac{1}{x^5}$$

$$= (x^2 + \frac{1}{x^2}) (x^3 + \frac{1}{x^3}) - (x + \frac{1}{x})$$

$$= \left\{ (x + \frac{1}{x})^2 - 2 \cdot x \cdot \frac{1}{x} \right\} \left\{ (x + \frac{1}{x})^3 - 3 \cdot x \cdot \frac{1}{x} (x + \frac{1}{x}) \right\} - (x + \frac{1}{x})$$

$$= \left\{ (6)^2 - 2 \right\} \left\{ (6)^3 - 3 \times 6 \right\} - 6$$

$$= (36 - 2) (216 - 18) - 6$$

$$= (34 \times 198) - 6$$

$$= 6732 - 6$$

$$= 3726$$

$$= \text{R.H.S}$$

(প্রমাণিত)

১৬ নং প্রশ্ন উত্তর (ক)

ধরি,

বিনামূল্যে সংখ্যাটি x

$$\therefore x^2 = 5x - 1$$

$$\text{বা. } x^2 + 1 = 5x$$

$$\text{বা. } \frac{x^2 + 1}{x} = \frac{5x}{x}$$

$$\text{বা. } \frac{x^2}{x} + \frac{1}{x} = 5$$

$$\therefore x + \frac{1}{x} = 5$$

(shown)

১৬ নং প্রশ্ন উত্তর (খ)

ক নং হতে প্রাপ্ত,

$$x + \frac{1}{x} = 5$$

আমরা জানি,

$$(x - \frac{1}{x})^2 = (x + \frac{1}{x})^2 - 4 \cdot x \cdot \frac{1}{x}$$

$$\text{বা. } x - \frac{1}{x} = \sqrt{(x + \frac{1}{x})^2 - 4}$$

$$= \sqrt{(5)^2 - 4}$$

$$= \sqrt{25 - 4}$$

$$= \sqrt{21}$$

প্রদত্ত রাশিঃ $x^3 - \frac{1}{x^3}$

$$= \left(x - \frac{1}{x}\right)^3 + 3 \cdot x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right)$$

$$= (\sqrt{21})^3 + 3\sqrt{21}$$

$$= 21\sqrt{21} + 3\sqrt{21}$$

$$= 24\sqrt{21}$$

∴

২৮ নং প্রশ্নে উত্তর (৬)

'ক' থেকে পাই,

$$x + \frac{1}{x} = 5$$

$$\text{L.H.S} = x^5 + \frac{1}{x^5}$$

$$= \left\{ \left(x^2 + \frac{1}{x^2}\right) \left(x^3 + \frac{1}{x^3}\right) \right\} - \left(x + \frac{1}{x}\right)$$

$$= \left\{ \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} \right\} \left\{ \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \right\} - \left(x + \frac{1}{x}\right)$$

$$= \left\{ (5)^2 - 2 \right\} \left\{ (5)^3 - 3 \times 5 \right\} - 5$$

$$= (25 - 2) (125 - 15) - 5$$

$$= 23 \times 110 - 5$$

$$= 2530 - 5$$

$$= 2525$$

$$= R.H.S$$

(Proved)

২৭ নং প্রশ্নের উত্তর (ক)

দেওয়া আছে,

$$a = \sqrt{6} + \sqrt{5}$$

$$\therefore \frac{1}{a} = \frac{1}{\sqrt{6} + \sqrt{5}}$$

$$= \frac{\sqrt{6} - \sqrt{5}}{(\sqrt{6} + \sqrt{5})(\sqrt{6} - \sqrt{5})}$$

$$= \frac{\sqrt{6} - \sqrt{5}}{(\sqrt{6})^2 - (\sqrt{5})^2}$$

$$= \frac{\sqrt{6} - \sqrt{5}}{6 - 5}$$

$$= \sqrt{6} - \sqrt{5}$$

A:

২৭ নং প্রশ্ন দেখাও

দেওয়া আছে,

$$a = \sqrt{6} + \sqrt{5}$$

কনং হতে প্রাপ্ত,

$$\frac{1}{a} = \sqrt{6} - \sqrt{5}$$

$$\begin{aligned} \therefore a + \frac{1}{a} &= \sqrt{6} + \sqrt{5} + \sqrt{6} - \sqrt{5} \\ &= 2\sqrt{6} \end{aligned}$$

প্রকৃত রাশি: $a^3 + \frac{1}{a^3}$

$$= \left(a + \frac{1}{a}\right)^3 - 3 \cdot a \cdot \frac{1}{a} \left(a + \frac{1}{a}\right)$$

$$= (2\sqrt{6})^3 - 3 \times 2\sqrt{6}$$

$$= (8 \times 6\sqrt{6}) - 6\sqrt{6}$$

$$= 48\sqrt{6} - 6\sqrt{6}$$

$$= 42\sqrt{6}$$

∴

১৭ নং প্রশ্নের উত্তর (৩)

১৫ নং হতে প্রাপ্ত,

$$a + \frac{1}{a} = 2\sqrt{6}$$

$$\text{১৬ নং } a^3 + \frac{1}{a^3} = 42\sqrt{6}$$

প্রদত্ত রাশিঃ $a^6 + \frac{1}{a^6}$

$$= (a^3)^2 + \left(\frac{1}{a^3}\right)^2$$

$$= \left(a^3 + \frac{1}{a^3}\right)^2 - 2 \cdot a^3 \cdot \frac{1}{a^3}$$

$$= (42\sqrt{6})^2 - 2$$

$$= 10584 - 2$$

$$= 10582$$

A:

২৮ নং প্রশ্ন উত্তর (ক)

কওয়া আছে,

$$x^4 - x^2 + 1 = 0$$

$$\text{বা, } x^4 + 1 = x^2$$

$$\text{বা, } \frac{x^4 + 1}{x^2} = \frac{x^2}{x^2}$$

$$\text{বা, } \frac{x^4}{x^2} + \frac{1}{x^2} = 1$$

$$\text{বা, } x^2 + \frac{1}{x^2} = 1$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} = 1$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 = 1 + 2$$

$$\text{বা, } \left(x + \frac{1}{x}\right)^2 = 3$$

$$\text{বা, } \left(x + \frac{1}{x}\right) = \sqrt{3}$$

$$\therefore x + \frac{1}{x} = \sqrt{3}$$

∴

২৮ নং প্রশ্নের উত্তর (খ)

ক নং হতে পাই,

$$x + \frac{1}{x} = \sqrt{3}$$

$$\text{L.H.S} = \frac{x^6 + 1}{x^3}$$

$$= \frac{x^6}{x^3} + \frac{1}{x^3}$$

$$= x^3 + \frac{1}{x^3}$$

$$= (x + \frac{1}{x})^3 - 3 \cdot x \cdot \frac{1}{x} (x + \frac{1}{x})$$

$$= (\sqrt{3})^3 - 3 \times \sqrt{3}$$

$$= 3\sqrt{3} - 3\sqrt{3}$$

$$= 0$$

$$= \text{R.H.S}$$

(সেখানে হল)

২৮ নং পূর্ণ উত্তর গা

দেওয়া আছে,

$$x + \frac{1}{x} = \sqrt{3}$$

$$\text{L.H.S} = x^5 + \frac{1}{x^5}$$

$$= (x^2 + \frac{1}{x^2}) (x^3 + \frac{1}{x^3}) - (x + \frac{1}{x})$$

$$= \left\{ (x + \frac{1}{x})^2 - 2 \cdot x \cdot \frac{1}{x} \right\} \times 0 - \sqrt{3}$$

$$= \left\{ (\sqrt{3})^2 - 2 \right\} \times 0 - \sqrt{3}$$

[যদিও হতে পারে
 $x^3 + \frac{1}{x^3} = 0$]

$$= (3-2) \times 0 - \sqrt{3}$$

$$= 1 \times 0 - \sqrt{3}$$

$$= 0 - \sqrt{3}$$

$$= -\sqrt{3}$$

$$= \text{R.H.S}$$

(Proved)

১নং প্রশ্নের উত্তর (ক)

দেওয়া আছে,

$$x^2 - \sqrt{5}x + 1 = 0$$

বা. $x^2 + 1 = \sqrt{5}x$

বা. $\frac{x^2 + 1}{x} = \frac{\sqrt{5}x}{x}$

বা. $\frac{x^2}{x} + \frac{1}{x} = \sqrt{5}$

$\therefore x + \frac{1}{x} = \sqrt{5}$

এ:

২নং প্রশ্নের উত্তর (খ)

ক নং হতে প্রাপ্ত,

$$x + \frac{1}{x} = \sqrt{5}$$

বা. $(x + \frac{1}{x})^2 = (\sqrt{5})^2$ [কর্গ করার]

বা. $x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = 5$

বা. $x^2 + 2 + \frac{1}{x^2} = 5$

বা. $x^2 + \frac{1}{x^2} = 5 - 2$

বা. $x^2 + \frac{1}{x^2} = 3$

$$\text{বা. } (x^2 + \frac{1}{x^2})^2 = (3)^2 \quad [\text{বর্গ করে}]$$

$$\text{বা. } (x^2)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + (\frac{1}{x^2})^2 = 9$$

$$\text{বা. } x^4 + 2 + \frac{1}{x^4} = 9$$

$$\text{বা. } x^4 + \frac{1}{x^4} = 9 - 2$$

$$\text{বা. } (x^4 + \frac{1}{x^4}) = 7$$

$$\text{বা. } (x^4 + \frac{1}{x^4})^2 = (7)^2 \quad [\text{পুনরায় বর্গ}]$$

$$\text{বা. } (x^4 - \frac{1}{x^4})^2 + 4 \cdot x^4 \cdot \frac{1}{x^4} = 49$$

$$\text{বা. } (x^4 - \frac{1}{x^4})^2 + 4 = 49$$

$$\text{বা. } (x^4 - \frac{1}{x^4})^2 = 49 - 4$$

$$\text{বা. } (x^4 - \frac{1}{x^4})^2 = 45$$

$$\text{বা. } x^4 - \frac{1}{x^4} = \sqrt{45}$$

$$\therefore x^4 - \frac{1}{x^4} = \sqrt{45}$$

∴

২২নং প্রশ্নের উত্তর (গ)

ক নং হতে প্রাপ্ত,

$$x + \frac{1}{x} = \sqrt{5}$$

$$\text{L.H.S} = x^5 + \frac{1}{x^5}$$

$$= \left\{ \left(x^2 + \frac{1}{x^2} \right) \left(x^3 + \frac{1}{x^3} \right) \right\} - \left(x + \frac{1}{x} \right)$$

$$= \left\{ \left(x + \frac{1}{x} \right)^2 - 2 \cdot x \cdot \frac{1}{x} \right\} \left\{ \left(x + \frac{1}{x} \right)^3 - 3 \cdot x \cdot \frac{1}{x} \right\} - \left(x + \frac{1}{x} \right)$$

$$= \left\{ (\sqrt{5})^2 - 2 \right\} \left\{ (\sqrt{5})^3 - 3 \times \sqrt{5} \right\} - \sqrt{5}$$

$$= (5 - 2) (5\sqrt{5} - 3\sqrt{5}) - \sqrt{5}$$

$$= 3 \times 2\sqrt{5} - \sqrt{5}$$

$$= 6\sqrt{5} - \sqrt{5}$$

$$= 5\sqrt{5}$$

$$= \text{R.H.S}$$

(Proved)

২৪ নং প্রশ্ন উত্তর (ক)

দেওয়া আছে,

$$a = \sqrt{6} + \sqrt{5}$$

$$\therefore \frac{1}{a} = \frac{1}{\sqrt{6} + \sqrt{5}}$$

$$= \frac{\sqrt{6} - \sqrt{5}}{(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5})}$$

$$= \frac{\sqrt{6} - \sqrt{5}}{(\sqrt{6})^2 - (\sqrt{5})^2}$$

$$= \frac{\sqrt{6} - \sqrt{5}}{6 - 5}$$

$$= \sqrt{6} - \sqrt{5}$$

A:

২০ নং প্রশ্ন উত্তর (খ)

দেওয়া আছে,

$$a = \sqrt{6} + \sqrt{5}$$

ক নং হতে প্রাপ্ত,

$$\frac{1}{a} = \sqrt{6} - \sqrt{5}$$

$$\begin{aligned} \therefore a - \frac{1}{a} &= \sqrt{6} + \sqrt{5} - \sqrt{6} + \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

প্রকঃ $a + \frac{1}{a} = \sqrt{6} + \sqrt{5} + \sqrt{6} - \sqrt{5}$

$$= 2\sqrt{6}$$

প্রদত্ত রাশি : $(a^2 - \frac{1}{a^2})(a^3 - \frac{1}{a^3})$

$$\begin{aligned} &= (a + \frac{1}{a})(a - \frac{1}{a}) \left\{ (a - \frac{1}{a})^3 + 3a \cdot \frac{1}{a} (a - \frac{1}{a}) \right\} \\ &= (2\sqrt{5} \times 2\sqrt{6}) \left\{ (2\sqrt{5})^3 + 3 \times 2\sqrt{5} \right\} \\ &= 4\sqrt{30} (40\sqrt{5} + 6\sqrt{5}) \\ &= 4\sqrt{30} \times 46\sqrt{5} \\ &= 184\sqrt{30 \times 5} \\ &= 184\sqrt{150} \\ &= 184\sqrt{6 \times 25} \\ &= 184 \times 5\sqrt{6} \\ &= 920\sqrt{6} \end{aligned}$$

A:

২০নং প্রশ্ন উত্তর (গ)

খ থেকে প্ৰাপ্ত,

$$a - \frac{1}{a} = 2\sqrt{5}$$

$$\text{প্রকং } a + \frac{1}{a} = 2\sqrt{6}$$

$$\text{L.H.S} = \frac{a^{12} - 1}{a^6}$$

$$= \frac{a^{12}}{a^6} - \frac{1}{a^6}$$

$$= a^6 - \frac{1}{a^6}$$

$$= (a^3)^4 - \left(\frac{1}{a^3}\right)^4$$

$$= (a^3 - \frac{1}{a^3})(a^3 + \frac{1}{a^3})$$

$$= \left\{ (a - \frac{1}{a})^3 + 3 \cdot a \cdot \frac{1}{a} (a - \frac{1}{a}) \right\}$$

$$\left\{ (a + \frac{1}{a})^3 - 3 \cdot a \cdot \frac{1}{a} (a + \frac{1}{a}) \right\}$$

$$= \left\{ (2\sqrt{5})^3 + 3 \times 2\sqrt{5} \right\} \left\{ (2\sqrt{6})^3 - 3 \times 2\sqrt{6} \right\}$$

$$= \left\{ (8 \times 5 \sqrt{5}) + 6\sqrt{5} \right\} \left\{ (8 \times 6 \sqrt{6}) - 6\sqrt{6} \right\}$$

$$= (40\sqrt{5} + 6\sqrt{5})(48\sqrt{6} - 6\sqrt{6})$$

$$= 46\sqrt{5} \times 42\sqrt{6}$$

$$= 1932 \sqrt{5 \times 6}$$

$$= 1932 \sqrt{30}$$

$$= \text{R.H.S}$$

২০নং প্রশ্ন উত্তর (ক)

দেওয়া আছে,

$$p^2 - 1 = 4p$$

$$\text{বা, } \frac{p^2 - 1}{p} = \frac{4p}{p}$$

$$\text{বা, } \frac{p^2}{p} - \frac{1}{p} = 4$$

$$\text{বা, } p - \frac{1}{p} = 4$$

$$\begin{aligned} \therefore (p + \frac{1}{p})^2 &= (p - \frac{1}{p})^2 + 4 \cdot p \cdot \frac{1}{p} \\ &= (4)^2 + 4 \\ &= 16 + 4 \\ &= 20 \end{aligned}$$

A.

২০ নং প্রশ্ন উত্তর (খ)

দেওয়া আছে,

$$p^2 - 1 = 4p$$

$$\text{পূঙ্জিত রাশি : } \frac{p^3 + 5p}{p^4 + 4p^2 - 5} \quad \sqrt[3]{64}$$

$$= \frac{P(P^2+5)}{(P^2)^2 + 2 \cdot P^2 \cdot 5 + 5^2 - 6P^2 - 30} \times \sqrt[3]{4^3}$$

$$= \frac{P(P^2+5)}{(P^2+5)^2 - 6(P^2+5)} \times 4$$

$$= \frac{P(P^2+5)}{(P^2+5)^2 - (P^2+5-6)} \times 4$$

$$= \frac{P(P^2+5)}{(P^2+5)(P^2-1)} \times 4$$

$$= \frac{P}{P^2-1} \times 4$$

$$= \frac{P}{4P} \times 4$$

$$= 1$$

A:

২০ক° পূর্ণা ত্রৈকোণ

'ক' থেকে পাই,

$$(p + \frac{1}{p})^2 = 20$$

$$\text{বা, } p^2 + 2 \cdot p \cdot \frac{1}{p} + \frac{1}{p^2} = 20$$

$$\text{বা, } p^2 + 2 + \frac{1}{p^2} = 20$$

$$\text{বা, } p^2 + \frac{1}{p^2} = 20 - 2$$

$$\text{বা, } p^2 + \frac{1}{p^2} = 18$$

$$\text{বা, } (p^2 + \frac{1}{p^2})^2 = (18)^2 \quad [\text{বর্গ করে}]$$

$$\text{বা, } (p^2)^2 + 2 \cdot p^2 \cdot \frac{1}{p^2} + (\frac{1}{p^2})^2 = 324$$

$$\text{বা, } p^4 + 2 + \frac{1}{p^4} = 324$$

$$\text{বা, } p^4 + \frac{1}{p^4} = 324 - 2$$

$$\text{বা, } p^4 + \frac{1}{p^4} = 322$$

$$\therefore p^4 = 322 - \frac{1}{p^4}$$

(প্রমাণিত)

২২ নং প্রশ্নের উত্তর (ক)

দেওয়া আছে,

$$p+q=6$$

$$pq=3 \quad \text{যেখানে } p>q$$

আমরা জানি,

$$(p-q)^2 = (p+q)^2 - 4pq$$

$$\text{বা. } p-q = \sqrt{(p+q)^2 - 4pq}$$

$$= \sqrt{(6)^2 - 4 \times 3}$$

$$= \sqrt{36 - 12}$$

$$= \sqrt{24}$$

$$= \sqrt{4 \times 6}$$

$$= 2\sqrt{6}$$

∴

২২নং প্রশ্নের উত্তর (খ)

দেওয়া আছে,

$$p+q = 6$$

$$pq = 3$$

$$p-q = 2\sqrt{6}$$

প্রদত্ত রাশিঃ $p^3 - q^3 - 5(p^2 - q^2)$

$$= (p-q)^3 + 3pq(p-q) - 5(p+q)(p-q)$$

$$= (2\sqrt{6})^3 + 3 \times 3 \times 2\sqrt{6} - 5 \times 6 \times 2\sqrt{6}$$

$$= 8 \times 6\sqrt{6} + 18\sqrt{6} - 60\sqrt{6}$$

$$= 48\sqrt{6} + 18\sqrt{6} - 60\sqrt{6}$$

$$= 66\sqrt{6} - 60\sqrt{6}$$

$$= 6\sqrt{6}$$

∴

২২নং প্রশ্নের (গ)

'ক' থেকে পাই,

$$P+Q = \sqrt{6}$$

$$P-Q = 2\sqrt{6}$$

$$P \cdot Q = 3$$

$$\text{L.H.S} = P^5 + Q^5$$

$$= (P^3 + Q^3)(P^2 + Q^2) - (P+Q)P^2Q^2$$

$$= \{ (P+Q)^3 - 3PQ(P+Q) \} \{ (P+Q)^2 - 2PQ \} - (P+Q)P^2Q^2$$

$$= \{ (\sqrt{6})^3 - 3 \times 3 \times \sqrt{6} \} \{ (\sqrt{6})^2 - 2 \times 3 \} - (\sqrt{6}) \times (3)^2$$

$$= (216 - 54)(36 - 6) - (6 \times 9)$$

$$= (162 \times 30) - 54$$

$$= 4860 - 54$$

$$= 4806$$

$$= \text{R.H.S}$$

(shown)